

D-Brane Boundary State Dynamics

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Abstract

We construct the open string boundary states corresponding to various time-dependent deformations of the D-brane and explore several ways in which they may be used to study stringy soliton collective coordinate quantum dynamics. Among other things, we find that D-strings have exact moduli corresponding to arbitrary chiral excitations of the basic soliton. These are presumably the duals of the BPS-saturated excitations of the fundamental Type IIB string. These first steps in a systematic study of the dynamics and interactions of Dirichlet-brane solitons give further evidence of the consistency of Polchinski's new approach to string soliton physics.

1. Introduction

Polchinski's remarkable proposal [1] that the R-R charge-carrying solitonic states of Type II string theory can be given an exact conformal field theory description via open strings with Dirichlet boundary conditions [2] has opened up a new chapter in the development of string theory. The problem of the proper description of solitons in string theory has always been fascinating, if elusive, but it has recently become particularly urgent with the realization that apparently different pairs of string theories are dual to each other, with the solitons of one being the fundamental strings of the other [3]. Polchinski suggests that, in order to obtain a full string theory description of many solitons, rather than look for some complicated conformal field theory of an extended spacetime object, it suffices to consider an open string subject to just enough Dirichlet boundary conditions to localize the center of mass of the soliton. This proposal, shocking in its simplicity (at least to anyone who has struggled with the more conventional approach to string solitons), has already passed quite a few consistency tests bearing mainly on static properties of the solitons [4,5,6]. In this paper, we examine how the new proposal deals with some issues in soliton dynamics, *viz.* collective coordinate quantization, scattering and excitation of internal degrees of freedom.

Once the soliton problem is recast as a problem in open string dynamics, the technical issue becomes that of finding an appropriate conformally invariant “boundary state” [7] and extracting target space information, such as the soliton mass, from it. As an aside, we note that one of the more puzzling old-style string soliton problems was to find a conformal field theory construction of the soliton mass: It is not the central charge and the correct construction, discovered only recently, is quite subtle [8]. Open string boundary conformal field theories, however, do define a quantity analogous to the central charge, known as the zero temperature boundary entropy [9], which has all the right scaling properties to be a target space action density. Polchinski's merging of solitons with open strings empowers us to make such an identification [10] and one of our concerns will be to show how it works in dynamical detail.

More generally, in this paper we will construct the boundary states which describe solitons in various states of collective coordinate excitation and will use these objects to extract as much dynamical information as we can.

2. Moving the D-Brane Boundary State

In order to endow a general Dirichlet D-brane with interesting dynamics one introduces the following boundary action [2,5,4]:

$$S_b = \oint d\sigma \left[\sum_{i=0}^p A_i(X^0, \dots, X^p) \frac{\partial}{\partial \sigma} X^i + \sum_{j=p+1}^9 \phi_j(X^0, \dots, X^p) \frac{\partial}{\partial \tau} X^j \right] .$$

We have suppressed the fermionic terms associated with world sheet supersymmetry and we have taken the boundary to lie at constant τ . The A_i are gauge fields on the D-brane world volume which need to be turned on if we wish to give it some NS-NS charge. The fields ϕ_j , on the other hand, are scalars from the world volume point of view: they describe transverse motions of the D-brane. The boundary gauge fields are of course not arbitrary: for string theory consistency, they must define a boundary conformal field theory. In this paper we will consider some very simple, physically interesting, special choices which are manifestly conformal. While the above action has no symmetry between X^i , $i = 0, \dots, p$ and X^j , $j = p+1, \dots, 9$, the symmetry is restored if we perform a T-duality transformation on X^j . This is why the D-brane dynamics has some hidden simplicity.

For definiteness we will construct boundary states for charged moving 1-branes, although our techniques may be generalized to any D-brane. For the 1-brane, the coordinates X^0 and X^1 , as well as their fermionic partners, have Neumann boundary conditions, while X^j ($j = 2, \dots, 9$) and their fermionic partners have Dirichlet boundary conditions. The NS-NS part of the stationary uncharged membrane's boundary state is,

$$|B_0\rangle = \prod_{j=2}^9 \delta(X^j) |B_\alpha\rangle |B_\psi\rangle |B_{\text{gh}}\rangle$$

The explicit expressions for the factors can be obtained from the results of [7] by applying the T-duality transformation needed to impose the appropriate Dirichlet boundary conditions:

$$\begin{aligned} |B_\alpha\rangle &= \exp \left\{ \sum_{n=1}^{\infty} \frac{1}{n} (\tilde{\alpha}_{-n}^0 \alpha_{-n}^0 - \tilde{\alpha}_{-n}^1 \alpha_{-n}^1 + \tilde{\alpha}_{-n}^j \alpha_{-n}^j) \right\} |0\rangle , \\ |B_\psi\rangle &= \exp \left\{ \sum_{n>0} (\tilde{\psi}_{-n}^0 \psi_{-n}^0 - \tilde{\psi}_{-n}^1 \psi_{-n}^1 + \tilde{\psi}_{-n}^j \psi_{-n}^j) \right\} |0\rangle , \\ |B_{\text{gh}}\rangle &= \exp \left\{ \sum_{n>0} (\tilde{\gamma}_{-n} \beta_{-n} - \gamma_{-n} \tilde{\beta}_{-n}) + \sum_{m=1}^{\infty} (c_{-m} \tilde{b}_{-m} + \tilde{c}_{-m} b_{-m}) \right\} |Z\rangle , \end{aligned}$$

where $|Z\rangle$ is the appropriate ghost vacuum. The mode numbers of the fermions and the superghosts are half-odd-integral.

Now introduce an electric field in the 1-direction, and also give the D-brane some transverse velocity in the 2-direction. To this end we introduce the boundary interaction

$$\oint d\sigma \left[EX^0 \frac{\partial}{\partial \sigma} X^1 + VX^0 \frac{\partial}{\partial t} X^2 \right] ,$$

together with appropriate fermionic terms required by the world sheet supersymmetry. The primary effect of this quadratic interaction is to produce a Lorentz boost on the left-moving part of the boundary state relative to the right-moving part. A secondary, but crucial, effect of turning on the boundary gauge fields A_i and ϕ_j is the appearance of an overall normalizing factor in front of the boundary state. As shown in [11,7], this factor is the Born-Infeld action for the boundary gauge fields. In the case at hand, where the boundary field strengths are $F_{01} = E$, $F_{02} = V$, the Born-Infeld normalizing factor reduces to $\sqrt{1 - E^2 - V^2}$ (the Lorentz signature of the metric is of course crucial here). The Born-Infeld action term is not affected by the Dirichlet boundary conditions for some of the fields [2], but the operator structure of the boundary state does suffer a rather trivial modification which we will make explicit and comment on shortly.

The explicit form of the boosted boundary state generated by turning on both E and V is

$$|B_{E,V}\rangle = \sqrt{1 - E^2 - V^2} \exp(O) |B_0\rangle$$

where $O = O_\psi + O_\alpha$ with

$$O_\alpha = 2\delta \left[\frac{E}{\sqrt{E^2 + V^2}} \sum_{n>0} \frac{1}{n} (\alpha_{-n}^0 \alpha_n^1 - \alpha_n^0 \alpha_{-n}^1) + \frac{V}{\sqrt{E^2 + V^2}} \sum_{n>0} \frac{1}{n} (\alpha_{-n}^0 \alpha_n^2 - \alpha_n^0 \alpha_{-n}^2) \right]$$

and O_ψ generates the same boost on the fermions. The hyperbolic angle δ is related to the “velocity” (E, V) in the usual manner of Lorentz transformations: $\cosh \delta = 1/\sqrt{1 - E^2 - V^2}$. The net effect is a boost of the left-movers relative to the right-movers. This expression for the boundary state reveals the interesting fact that, while the boundary state is not $SO(1, 9)$ invariant, the operator that generates it from the “bare” boundary state is an element of $SO(1, 9)$.

Consider first the simple special case $V = 0$ (so that $\cosh \delta = 1/\sqrt{1 - E^2}$). The dynamic boundary state is constructed from the static one by replacing

$$\begin{pmatrix} \alpha^0 \\ \alpha^1 \end{pmatrix} \rightarrow \begin{pmatrix} \cosh(2\delta) & \sinh(2\delta) \\ \sinh(2\delta) & \cosh(2\delta) \end{pmatrix} \begin{pmatrix} \alpha^0 \\ \alpha^1 \end{pmatrix} , \quad \begin{pmatrix} \psi^0 \\ \psi^1 \end{pmatrix} \rightarrow \begin{pmatrix} \cosh(2\delta) & \sinh(2\delta) \\ \sinh(2\delta) & \cosh(2\delta) \end{pmatrix} \begin{pmatrix} \psi^0 \\ \psi^1 \end{pmatrix}$$

and multiplying by the Born-Infeld factor $\sqrt{1-E^2}$. In terms of E , the boost matrix is

$$\begin{pmatrix} \alpha^0 \\ \alpha^1 \end{pmatrix} \rightarrow \frac{1}{1-E^2} \begin{pmatrix} 1+E^2 & 2E \\ 2E & 1+E^2 \end{pmatrix} \begin{pmatrix} \alpha^0 \\ \alpha^1 \end{pmatrix} .$$

By expanding the boundary state in creation operators, one can read off that the source for the antisymmetric tensor B_{01} is $2E/\sqrt{1-E^2}$, while the graviton source strength is $(1+E^2)/\sqrt{1-E^2}$. In the following sections, we will see that various conclusions about soliton dynamics can be read off from these results.

3. Quantizing the Born-Infeld Action

In order to relate E to the NS-NS charge and to identify the energy per unit length of the string, we need an action. We will make the plausible guess that the normalization factor for the boundary state (the same thing as the disk amplitude) can be taken as the action for the collective coordinate $A_1(t)$ and quantized. To give the string a definite length, let us compactify X^1 on a circle of length l . Now the Born-Infeld Lagrangian collapses to

$$L = -\frac{l}{\lambda} \sqrt{1 - \dot{A}_1^2} , \quad \dot{A}_1 = E .$$

For completeness, we have introduced the power of the string coupling constant λ appropriate to the origin of this action in the disk amplitude (had the action somehow been derived from a sphere amplitude, the correct power would have been λ^{-2}).

A subtle but crucial feature of the theory is that the presence of large gauge transformations makes A_1 a compact variable with period $2\pi/l$. Therefore, the momentum conjugate to A_1 ,

$$Q = \frac{l\dot{A}_1}{\lambda\sqrt{1 - \dot{A}_1^2}} ,$$

is quantized in units of l .¹ The Hamiltonian derived from L is

$$H = Q\dot{A}_1 - L = \sqrt{\left(\frac{l}{\lambda}\right)^2 + Q^2} = l\sqrt{\left(\frac{1}{\lambda}\right)^2 + n^2} ,$$

where $n = Q/l$ is an integer. Remarkably, the string energy per unit length is precisely the BPS formula for the tension of $(n, 1)$ strings [12,5,4]! The boundary state can now be

¹ Our definition of the charge differs from that in [4].

rewritten in terms of the charge, rather than the less directly meaningful field strength E and we find, for example, that the source for B_{01} is $2n$, which is proportional to the NS-NS charge per unit length.

We can give a similar treatment to the general case of a charged, moving 1-brane. Here both E and V are non-vanishing and the boundary state is defined by a more complicated Lorentz transformation, one which mixes the longitudinal and transverse directions:

$$\begin{pmatrix} \alpha^0 \\ \alpha^1 \\ \alpha^2 \end{pmatrix} \rightarrow \frac{1}{1 - E^2 - V^2} \begin{pmatrix} 1 + E^2 + V^2 & 2E & 2V \\ 2E & 1 - V^2 + E^2 & 2VE \\ 2V & 2VE & 1 - E^2 + V^2 \end{pmatrix} \begin{pmatrix} \alpha^0 \\ \alpha^1 \\ \alpha^2 \end{pmatrix} \quad (3.1)$$

and similarly for the left-moving fermion modes. The Born-Infeld action normalizing factor, which we want to treat as the effective lagrangian for the collective coordinates, now turns out to be

$$L = -\frac{l}{\lambda} \sqrt{1 - \dot{A}_1^2 - \dot{X}_\perp^2}$$

and the Hamiltonian is

$$H = \sqrt{\left(\frac{l}{\lambda}\right)^2 + Q^2 + P_\perp^2}$$

where

$$Q = \frac{l\dot{A}_1}{\lambda\sqrt{1 - \dot{A}_1^2 - \dot{X}_\perp^2}}, \quad P = \frac{l\dot{X}_\perp}{\lambda\sqrt{1 - \dot{A}_1^2 - \dot{X}_\perp^2}}.$$

The string energy per unit length is

$$\sqrt{\left(\frac{1}{\lambda}\right)^2 + n^2 + p_\perp^2}$$

where $p_\perp = P_\perp/l$ is the momentum density. This is just the relativistic expression for the energy density of a moving straight string! If we examine the boundary state appropriate to this case, we see that the source for B_{01} is again equal to $2n$. There is also a source for B_{12} equal to $-2Vn$, but no source for B_{02} .

Next, we might ask whether there is an action to quantize in order to obtain the tension of the multiply-wound (n, m) strings, where m refers to the R-R charge and n to the NS-NS charge. For the doubly-wound strings ($m = 2$), for instance, the BPS mass formula tells us that such states really exist only for n odd (the even-charged states are neutrally stable with respect to splitting into two singly-wound strings). Is there a generalization of the Born-Infeld action which allows us to examine this case too? Witten

has shown [5] that the proper setup for discussing the dynamics of multiple strings is a $N = 8$ supersymmetric $U(m)$ gauge theory in $1+1$ dimensions. He has further argued that there are certain vacua where the $SU(m)$ part of the theory develops a mass gap (they are described by placing n quarks at infinity such that m and n are relatively prime). A physical consequence of this mass gap is the formation of a bound state. If the $SU(m)$ part of the gauge field is frozen out, then we may replace the $U(m)$ gauge field \mathbf{A}_μ by $A_\mu I$, where I is the m -dimensional identity matrix. Now the relevant Born-Infeld lagrangian is

$$L = -\frac{lm}{\lambda} \sqrt{1 - \dot{A}_1^2} ,$$

where the factor of m comes from tracing over the $U(m)$ indices. Quantization of this $U(1)$ theory is analogous to what we encountered in the $m = 1$ case. All the formulae carry over with the replacement of $1/\lambda$ by m/λ . Thus, we find that the tension of (n, m) strings is

$$\sqrt{\left(\frac{m}{\lambda}\right)^2 + n^2} ,$$

Remarkably, this again agrees with the BPS formula! While this is very encouraging, we clearly need a better understanding of why the $SU(m)$ part of the theory may be ignored when m and n are relatively prime.

4. Forces On Moving D-Branes

Boundary states can be used to calculate the forces between D-branes. Roughly speaking, the annulus amplitude obtained by gluing two boundary states together gives the interaction energy due to closed string exchange. The first thing we would like to extract from such an exercise is that the force between separated, but otherwise identical, D-branes vanishes. This is the no-force condition on BPS saturated states and should follow from supersymmetric boson-fermion cancellations in the annulus amplitude. The requisite cancellations have been shown to occur for the basic soliton [1], and for static charged solitons [4]. We will verify that the force cancellation also occurs for charged, translating solitons. The same formalism allows us to calculate the force between solitons of different charges and/or non-zero relative velocity. In these cases the BPS saturation argument for vanishing force doesn't work and we indeed find a non-vanishing long-range force with an interesting dependence on charges and velocities.

This calculation brings the full boundary state, including both the NS-NS and R-R sectors, into play. This object was constructed in studies of the Fischler-Susskind mechanism in string theory [7] and Li has recently shown how to modify these old results to obtain the D-brane boundary state [4]. We will make heavy use of his results here.

The boundary states are a convenient tool for summing over the forces mediated by the NS-NS and the R-R bosons. For simplicity, we will restrict our attention to the long range force and project the boundary state onto the massless levels. In the picture with superghost charge -2 , the metric and the antisymmetric tensor components of that projection may be concisely written as

$$\sqrt{1 - E^2 - V^2} \tilde{\psi}_{-1/2}^\mu (\eta \cdot \Lambda_{E,V} \cdot T)_{\mu\nu} \psi_{-1/2}^\nu |Z\rangle$$

where $\Lambda_{E,V}$ is the ten-dimensional Lorentz transformation matrix whose non-trivial 3×3 corner is given in (3.1), $\eta = \text{diag}(-1, 1, \dots, 1)$ is the Minkowski metric and $T = \text{diag}(1, 1, -1, \dots, -1)$ is the net effect of the T-duality transformation which imposes the Dirichlet boundary condition on X^j , $j = 2, \dots, 9$. A ghost dilaton component

$$\sqrt{1 - E^2 - V^2} (\beta_{-1/2} \tilde{\gamma}_{-1/2} - \tilde{\beta}_{-1/2} \gamma_{-1/2}) |Z\rangle$$

must be added to this to obtain the full boundary state.

We will use these results to calculate the long-range force between two 1-branes carrying different charges and moving with different velocities. By inserting the closed string propagator between an (E_1, V_1) boundary state and an (E_2, V_2) boundary state, we find that the net interaction due to the NS-NS exchanges is equal to that between two $E = 0$ stationary 1-branes multiplied by the prefactor

$$\mathcal{P}_{NS} = \sqrt{1 - E_1^2 - V_1^2} \sqrt{1 - E_2^2 - V_2^2} [\text{tr}(\eta \Lambda_{E_2, V_2}^t \eta \Lambda_{E_1, V_1}) - 2] .$$

The basic interaction potential due to massless exchanges in eight transverse dimensions is r^{-6} , and we could, if needed, obtain its absolute normalization.

Space-time supersymmetry leads us to expect that, for two stationary $E = 0$ D-branes, the net NS-NS force is exactly cancelled by the net R-R force. The general R-R boundary state is quite awkward to construct, but there is a simple recipe for the projection of the constant gauge field strength boundary state onto the massless levels. We will just state the recipe and refer the interested reader to [7,4] for detailed justification. The first observation is that this projected boundary state involves only world sheet fermion zero modes (there

is not even a normalizing determinant factor: the bosonic boundary determinant exactly cancels its fermionic partner). The zero-mode part of the constant gauge field strength boundary state is represented by a polynomial in components of $F_{\mu\nu}$ with the spacetime indices saturated on zero modes $\theta_0^\mu = \psi_0^\mu + \tilde{\psi}_0^\mu$. Because of anticommutativity, no individual θ_0^μ can appear more than once in any monomial. For the case at hand we have

$$\mathcal{P}_R = 1 - E\theta_0^0\theta_0^1 - V\theta_0^0\theta_0^2 .$$

The three terms 1, $\theta_0^0\theta_0^1$ and $\theta_0^0\theta_0^2$ can be regarded as orthogonal states of unit normalization in computing inner products between two boundary states. It follows that the net force due to the R-R exchanges is equal to the corresponding force between two $E = V = 0$ 1-branes multiplied by $1 - E_1E_2 - V_1V_2$ (the minus signs come from the anticommutativity properties of the θ_0^μ). The overall normalization of the force will contain an important factor 8 which comes from the dimensionality of the spinor space on which the fermion zero modes act.

Adding up the NS-NS and the R-R forces we get the following prefactor for the total long-range force,

$$\mathcal{P}_{tot} = \sqrt{1 - E_1^2 - V_1^2} \sqrt{1 - E_2^2 - V_2^2} [\text{tr}(\eta \Lambda_{E_2, V_2}^t \eta \Lambda_{E_1, V_1}) - 2] - 8(1 - E_1E_2 - V_1V_2) . \quad (4.1)$$

Using the defining relation of Lorentz transformations, it is a simple matter to check that the total force between identical D-branes ($E_1 = E_2$ and $V_1 = V_2$) cancels. This is a consequence of BPS saturation which, in turn, is a consequence of the fact that the boundary state, even for non-zero E and V , is annihilated by half the supersymmetries. Evaluating (4.1) for general E and V gives

$$\mathcal{P}_{tot} = 4 \frac{2 - (E_1 + E_2)^2 - (V_1 + V_2)^2 + (E_1^2 + V_1^2)(E_2^2 + V_2^2) + (E_1E_2 + V_1V_2)^2}{\sqrt{1 - E_1^2 - V_1^2} \sqrt{1 - E_2^2 - V_2^2}} - 8(1 - E_1E_2 - V_1V_2) \quad (4.2)$$

For $E_1 = E_2$, this expression describes the velocity-dependent force between two identical D-branes (for which the static force vanishes by BPS saturation). The term $\sim (V_1 - V_2)^2$, in turn, is the metric on moduli space for the scattering of two D-branes. It is a simple calculation to show that, for $E_1 = E_2$ this vanishes, and the leading term in the force is $\sim (V_1 - V_2)^4$ (this agrees with Bachas's conclusion [13] that the metric on the moduli space of two 1-branes vanishes). Remarkably, the forces between long fundamental strings have

the same type of velocity dependence for small velocities [14]! This might serve as another argument in favor of the $SL(2, Z)$ duality relating the solitonic and fundamental strings [12]. Furthermore, (4.2) shows that for 1-branes with unequal NS-NS charges the metric on moduli space no longer vanishes (the same property holds for the long fundamental strings).

5. Waves on D-branes

Now that we have found a boundary CFT description of D-branes rigidly moving in transverse directions, it is interesting to look for their internal excitations. In the following we will work out a very interesting example of a 1-brane stretched around a compact dimension of length l . Generalizations to D-branes with $D > 1$ are possible and will be discussed briefly.

In order to describe a Dirichlet string stretched around a compact dimension, we identify $X^1 \sim X^1 + l$. A transverse excitation, polarized in the direction ϵ^j and left-moving along the string, is described by the boundary operator

$$\oint d\sigma \left[\sum_p a_p e^{ipX^+} \epsilon^j \left(\frac{\partial}{\partial \tau} X^j + ip\psi^+ \psi^j \right) + c.c. \right] \quad (5.1)$$

where we have introduced the light-cone components,

$$X^\pm = X^0 \pm X^1, \quad \psi^\pm = \psi^0 \pm \psi^1$$

Note that the compactness of X^1 restricts the allowed values of p to $2\pi n/l$, where n is an integer. The crucial feature of the operator above is that, due to the Minkowski signature of space-time, it has the marginal dimension 1 for any value of p .

We will now argue that a theory with the boundary operator (5.1) added to the action describes an exactly conformal theory for any a_p . The first step towards establishing this is the calculation of the disk partition function Z_{disk} . For instance, the term of order $(a_p a_p^*)^m$ in the expansion of $\ln Z_{\text{disk}}$ is given by the connected correlation function of $2m$ boundary operators. Remarkably, all the connected n -point functions of the operator (5.1) vanish for $n > 2$. This is because the two-point functions of the light-cone components obey

$$\langle X^+(\sigma_1) X^+(\sigma_2) \rangle = \langle \psi^+(\sigma_1) \psi^+(\sigma_2) \rangle = 0.$$

A simple way to confirm that the operator (5.1) is exactly marginal is by adopting the light-cone gauge $X^+ = \tau$. In this context the operator (5.1) implements a shift of the transverse coordinates which depends only on τ . Since the light-cone boundary state is constructed at fixed τ , the only effect on the light-cone boundary state is a shift in the zero mode, which implies that the theory remains free. The conclusion is that an *arbitrary* left-moving excitation of the Dirichlet 1-brane is a conformally invariant background.

The next step would be to show that this class of boundary conformal field theories can be extended to a full supersymmetric boundary state, annihilated by a linear combination of the left- and right-moving supersymmetry generators. That would show that these propagating wave deformations of the 1-brane are still BPS-saturated states. We know how to do this supersymmetric extension for the constant background gauge field case and we believe that it can be done here as well (we'll save the details for another paper). These states are presumably the duals of the known BPS-saturated excited states of an extended Type IIB fundamental string. The latter are constructed by applying purely left-moving, or purely right-moving oscillators to the fundamental string ground state and correspond to disturbances propagating in only one direction along the string. The left-right level matching condition then requires that the winding fundamental string be endowed with a non-vanishing longitudinal momentum in the winding direction ($p_R^2 - p_L^2 = N_L - N_R$ to be precise). There should be a corresponding condition on the 1-brane excitations, but we have yet to identify it.

Now we turn to boundary actions which describe a Dirichlet string carrying both left- and right-moving excitations:

$$\oint d\sigma \left[\sum_p a_p e^{ipX^+} \epsilon^j \left(\frac{\partial}{\partial \tau} X^j + ip\psi^+ \psi^j \right) + \sum_k \tilde{a}_k e^{ikX^-} \tilde{\epsilon}^j \left(\frac{\partial}{\partial \tau} X^j + ik\psi^- \psi^j \right) + c.c. \right] \quad (5.2)$$

It is easy to see that, unlike (5.1), in general this does not define a conformal field theory. Consider, for instance, the connected four-point function for the forward scattering of two right-movers and two left-movers. This is the coefficient of the $a_p a_p^* a_k a_k^*$ term in the disk amplitude. If this four-point function does not vanish, then the theory has a non-trivial beta function. Let us choose $\epsilon^j \tilde{\epsilon}_j = 0$, so that the right and left-movers are polarized at right angles. The four-point function is then given by the following integral,

$$\int_{-\infty}^{\infty} dt (|t|^{4pk} |1 - t|^{-4pk} - 1) - \int_{-\infty}^{\infty} dt |t|^{4pk} |1 - t|^{-4pk} \frac{4pk}{t(t-1)}$$

This integral can be calculated exactly and exhibits an infinite sequence of poles in the variable $4pk$. The origin of these poles, which are located at the odd integer values of $4pk$, may be traced to massive string states propagating along the 1-brane. Consider, for example, the OPE

$$e^{ipX^+} \frac{\partial}{\partial \tau} X^2(\sigma_1) e^{ikX^-} \frac{\partial}{\partial \tau} X^3(\sigma_2) \\ \sim |\sigma_1 - \sigma_2|^{-4pk} \frac{\partial}{\partial \tau} X^2 \frac{\partial}{\partial \tau} X^3 e^{i(p+k)X^0 + i(p-k)X^1} + \dots$$

This identifies the operator which gives rise to the first pole in the collision of a right-mover and a left-mover, located at $4pk = 1$.

Thus, we reach a conclusion that the Dirichlet strings differ from the fundamental strings in an essential way: while the latter are exactly described by the Nambu-Goto action, for the former it is at best an approximate low-energy effective description. The presence of the infinite sequence of poles indicates that there are extra degrees of freedom on the Dirichlet brane world volume which are not contained in the Nambu-Goto action. These new degrees of freedom are simply the massive modes of the open string whose ends are attached to the p -brane. The effects of these modes have already been observed in the scattering of massless particles off the p -branes [6]: they give rise to an infinite sequence of poles in the s -channel. The presence of such new soliton degrees of freedom raises the question whether the $SL(2, Z)$ symmetry, which interchanges fundamental strings and Dirichlet 1-branes is really an exact symmetry of the type IIB theory. If the $SL(2, Z)$ symmetry is exact, it requires that the fundamental string behavior changes very much as the coupling increases. Since the $SL(2, Z)$ relates fundamental strings at strong coupling to the Dirichlet strings at weak coupling, our results imply that the strongly coupled fundamental strings are not exactly described by the Nambu-Goto action, but acquire an infinite set of additional degrees of freedom. This result is not unexpected: because of loop corrections to string equations of motion the conformal field theory is not applicable to strongly coupled fundamental strings.

While the simultaneous presence of the left and right movers on the 1-brane in general destroys the world sheet conformal invariance, left and right-movers can coexist if the left and right-moving momenta are chosen in such a way as to eliminate logarithmic divergences in the disk partition function. Consider, for instance, the boundary action

$$\oint d\sigma A \left[e^{ipX^+} \epsilon^j \left(\frac{\partial}{\partial \tau} X^j + ip\psi^+ \psi^j \right) + e^{ipX^-} \epsilon^j \left(\frac{\partial}{\partial \tau} X^j + ip\psi^- \psi^j \right) + c.c. \right] \quad (5.3)$$

which describes a standing wave of amplitude A on the Dirichlet string. If $2p^2 = 1, 2, 3, \dots$, then the perturbative expansion of the disk partition function reduces to integrals of integer powers of $\sigma_i - \sigma_j$. As shown in [15], such integrals may be reduced to complex contour integrals enclosing at worst multiple pole singularities. As a result, they contain no logarithmic divergences and represent conformal fixed points. Thus, it appears that we have found a whole new family of boundary conformal field theories similar to the exact solutions of dissipative quantum mechanics obtained in [16]. The new boundary CFT's are especially interesting because they describe vibrational excitations of the Dirichlet 1-branes with arbitrarily large amplitude. An even more general class of boundary CFT's would describe the excitations of other D-branes, and we hope to give a detailed treatment of these theories in future publications.

6. Conclusions

In this paper we have shown how Polchinski's boundary state approach to dual string solitons can be extended to deal with the dynamical issues of zero mode quantization, soliton scattering and excitation of internal degrees of freedom. Everything we have found is consistent with generic physical expectations for the properties of solitonic extended objects. Precise predictions for the spectrum of soliton excitations can be obtained from BPS saturation and duality with extended fundamental strings. In the simplest case, namely the NS-NS charged excitations of the once-wound soliton string, we could compare these predictions with the the results of quantizing a zero-mode coordinate and found perfect agreement. Our overall conclusion is that Polchinski's Dirichlet brane proposal contains within it the seeds of a complete and consistent dynamics of dual solitons.

What has been done is just the tip of a large iceberg. The boundary states corresponding to most non-trivial excited states of the soliton have yet to be constructed in any detail. A lot remains to be done in that direction. Also, the conformal boundary state corresponds, roughly speaking, to the classical action and classical field configurations of the soliton. It is absolutely crucial to quantize this system, especially to explore duality issues, and we have shown how, making some plausible assumptions, that can be done in the simplest case. What's at issue more generally is the string theory analog of collective coordinate quantization. This is already a complicated subject in field theory and virtually nothing (with the exception of the interesting work reported in [8]) is known about how it works in string theory. Now that Polchinski has provided us with a manageable classical starting point for the discussion, perhaps progress can be made.

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References

- [1] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges”, NSF Institute for Theoretical Physics preprint, hep-th/9510017.
- [2] J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073; R. G. Leigh, Mod. Phys. Lett. A4 (1989) 2767.
- [3] C. M. Hull and P. K. Townsend, Nuc. Phys. B438 (1995) 109; E. Witten, Nuc. Phys. B443 (1995) 85; J. Schwarz, Lett. Math. Phys. 34 (1995) 309; A. Sen, Phys. Lett. B329 (1994) 217; J. A. Harvey and A. Strominger, Nuc. Phys. B449 (1995) 535.
- [4] M. Li, “Boundary States of D-Branes and Dy-Strings”, Brown University preprint, hep-th/9510161.
- [5] E. Witten, “Bound States of Strings and p-Branes”, IAS preprint, hep-th/9510135.
- [6] I. R. Klebanov and L. Thorlacius, “The Size of p-Branes”, Princeton preprint PUPT-1574, hep-th/9510200.
- [7] C. Callan, C. Nappi, C. Lovelace and S. Yost, Nuc. Phys. B308 (1988) 221.
- [8] W. Fischler, S. Paban and M. Rozali, Phys. Lett. B352 (1995) 298.
- [9] I. Affleck and A. W. W. Ludwig, Phys. Rev. Lett. **67** (1991) 161.
- [10] J. Polchinski, Phys. Rev. D50 (1994) 6041.
- [11] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. 163B (1985) 123; A. Abouelsaood, C. G. Callan, C. R. Nappi and S. A. Yost, Nucl. Phys. B280 (1987) 599.
- [12] J. Schwarz, “An $SL(2,Z)$ Multiplet of Type IIB Superstrings”. California Institute of Technology preprint, hep-th/9508143.
- [13] C. Bachas, “D-Brane Dynamics”, ITP and Ecole Polytechnique preprint, hep-th/9511043.
- [14] C. Callan, J. Maldacena and A. Peet, “Extremal Black Holes as Fundamental Strings”, Princeton preprint PUPT-1565, hep-th/9510134.
- [15] C. Callan, A. Felce and D. Freed, Nuc. Phys. B392 (1993) 551.
- [16] C. G. Callan and I. R. Klebanov, Phys. Rev. Lett. 72 (1994) 1968; C. G. Callan, I. R. Klebanov, A. W. W. Ludwig and J. Maldacena, Nuc. Phys. B422 (1994) 417; C. G. Callan, I. R. Klebanov, J. Maldacena and A. Yegulalp, Nuc. Phys. B443 (1995) 444.